Journal of Forestry Research (2013) 24(1): 15–22
DOI 10.1007/s11676-012-0270-4

ORIGINAL PAPER

Stem taper equations for poplars growing on farmland in Sweden

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Received: 2011-03-26; Accepted: 2011-11-22

Abstract: We developed a simple polynomial taper equation for poplars growing on former farmland in Sweden and also evaluated the performance of some well-known taper equations. In Sweden there is an increasing interest in the use of poplar. Effective management of poplar plantations for high yield production would be facilitated by taper equations providing better predictions of stem volume than currently available equations. In the study a polynomial stem taper equation with five parameters was established for individual poplar trees growing on former farmland. The outputs of the polynomial taper equation were compared with five published equations. Data for fitting the equations were collected from 69 poplar trees growing at 37 stands in central and southern Sweden (lat. 55-60° N). The mean age of the stands was 21 years (range 14-43), the mean density 984 stems·ha^{-1} (198–3,493), and the mean diameter at breast height (outside bark) 25 cm (range 12-40). To verify the tested equations, performance of accuracy and precision diameter predictions at seven points along the stem was closely analyzed. Statistics used for evaluation of the equations indicated that the variable exponent taper equation presented by Kozak (1988) performed best and can be recommended. The stem taper equation by Kozak (1988) recommended in the study is likely to be beneficial for optimising the efficiency and profitability of poplar plantation management. The constructed polynomial equation and the segmented equation presented by Max & Burkhart (1976) were second and third ranked. Due to the statistical complexity of Kozak's equation, the constructed polynomial equation is alternatively recommended when a simple model is requested and larger bias is accepted.

Key words: poplar; variable exponent taper equation; segmented model; simple taper equation; forest management

Foundation project: This work was financially supported by Skogssällskapet foundation.

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Introduction

Until recently planting of poplar trees in Sweden has been confined to small plantations, mostly established between 1980 and 1990 on set-aside farmland to assess their productivity. Poplar plantations on less than 500 ha in Sweden are older than ten years and most are dominated by the clone OP 42 (*Populus. maximowiczii* x *P. trichocarpa*). However, demand for biofuel in Sweden has increased the interest in poplar, among other species that are suitable for short rotations, hence stands on an additional 120 ha have been planted recently on former forest land where previous stands were damaged by wind during the storm Gudrun in 2005 (Rytter et al. 2011). The advantages of growing poplar as an exotic species in short rotation forestry have been discussed in several recent publications from a production perspective (Jonsson 2008; Christersson 2010), and several authors have considered ecological and environmental aspects of poplar plantations (Karacic 2005; Christersson & Verwijst 2006).

The terms 'form' and 'taper' are often used synonymously. In a paper entitled "The Form and Taper of Forest-Tree Stems" Gray (1956) provides clarification of the terms, with 'form' describing the shape or structure of the stem, e.g. a cone or paraboloid, whereas 'taper' is defined as 'the rate of narrowing in diameter in relation to increase in height of a given shape or form'. The expressions 'form factor' (the ratio of tree volume to the volume of a cylinder, of equal diameter to the breast height diameter of the tree) and 'slenderness' (DBH/H) provide a general indication of a tree's form or shape but do not provide any detail about how the diameter narrows with stem height increases, which can only be provided by a taper equation. Taper equations have been one of the most important topics of study in forest management (Fang & Bailey 1999). The equations are based on the diameter at breast height (DBH), total tree height (H) and height above ground to the measurement point (h) as independent variables and provide estimates of: stem diameter at any given stem height, total stem volume, merchantable volume and merchantable height to any top diameter and from any stump height, and individual log volumes of any length at any height from the ground (Kozak 2004). Many equations have been de-

veloped for different tree species.

There are two reasons for the importance of this area of study (Newnham 1988): no single theory has been able to explain satisfactorily all the variability in tree stem shape. Stem taper is a complex trait (Assmann 1970) that varies substantially depending on genetic factors (within- and among-species), environmental factors (*inter alia* soil type, hydrology, altitude and climate), forest management practices (Steven 1988; Karlsson 2005) and interactions between these factors. The range of factors involved (natural and anthropogenic) complicates the development of a universal model for tree stem taper. Taper equations have provided a flexible tool for estimating total and merchantable tree volume which can be used as market trends and product specifications change. From a practical point of view the latter reason is the most important (Muhairwe 1999).

According to Sterba (1980) many forms and types of stem taper equation have been published and evaluated. Models have been constructed to describe the taper of diverse species in various regions globally, based on equations of the following three types (Diéguez-Aranda et al. 2006; Sakici et al. 2008): (1) Simple taper equations (Kozak et al. 1969; Demaerschalk 1972; Demaerschalk 1973; Ormerod 1973; Sharma and Odervald 2001); (2) Segmented taper equations (Max & Burkhart 1976; Clark et al. 1991); (3) Variable exponent taper equations (Kozak 1988; Newnham 1992)

Until the middle of the 1970s simple equations have been presented (Figueiredo-Filho and Schaaf 1999). These equations were insufficient for describing the stem part near the base or at the top of the stem. Therefore, alternative models were proposed to solve the problem. Max and Burkhart (1976) proposed a segmented equation in which tree stem was divided into three sections (neiloid at base, paraboloid in middle and cone-shaped at top), which are represented by sub-functions connected at joint points for continuity. Variable exponent equations utilise an exponent that changes continuously along the stem, reflecting differences between the neiloid, paraboloid and cone-shaped sections (Kozak 1988; Newnham 1992). Variable exponent taper equations have been found to be superior to segmented and simple models in estimating stem diameters and volumes (Kozak 1988; Newnham 1992; Muhairwe 1999). However, variable exponent taper equations can't be integrated analytically to calculate total stem or log volumes (Diéguez-Arunda et al. 2006). The volume is estimated from a calculated diameter and length by numerical integration (Kozak 1988).

A number of variants of both segmented and variable exponent taper equations have been developed and applied, and the latter have been shown to exhibit less bias and have better predictive abilities than other models in several studies (Li et al. 2010; Sakici 2008). However, despite the advantages of these two model types they suffer from statistical complexity, and difficulties in estimating parameters and rearranging the models to calculate heights for given diameters. There is a need for simple equations in practical forest management. A simple equation developed by Kozak et al. (1969) has in various articles for different species been assessed in Brazil (Figueiredo-Filho and Schaaf 1999).

Tree volume and taper equations and yield tables have been developed for poplar (Benbrahim and Gavaland 2003) but their applicability to poplar stands in Sweden has not been assessed. Such tables and equations are important for formulating practical recommendations to support forest managers in estimating future biomass and volume yields of poplar plantations.

The objective of the present study was to develop and evaluate a simple polynomial taper equation for poplars growing on former farmland in Sweden and to evaluate the performance of some well-known taper equations. The selected equation should be suitable for practical use to support efficient, profitable management of poplar plantations.

Material and methods

Data

The predictive capabilities of the equations were compared by evaluating individual trees in poplar plantations on former farmland in central and southern Sweden. Most of the stands were planted between 1988 and 1992. The stands were established as research-sites, for commercial use with focus on production, or as demonstration sites. The sites cover a variety of site- and stand characteristics, Table 1. The water table was 0.3–1 m deep, and apart from a few sites with silty till soils, all other soils were clay sediments with textures ranging from light to medium clay. Data for constructing stem-taper equations were collected from 69 poplar trees growing at 37 stands in central and southern Sweden between latitudes 55–60° N (Fig. 1). The ages of the stands at the sites ranged between 14 and 43 years. The management of the stands varied; some had not been thinned at all and thinning regimes ranging from moderate to heavy thinning regimes had been applied in the others. The number of stems varied from 198 to 3,493 per hectare, which cover most of existing stand densities. In some stands the initial spacing and number of plants was known, but for most of the stands, these figures are unknown.

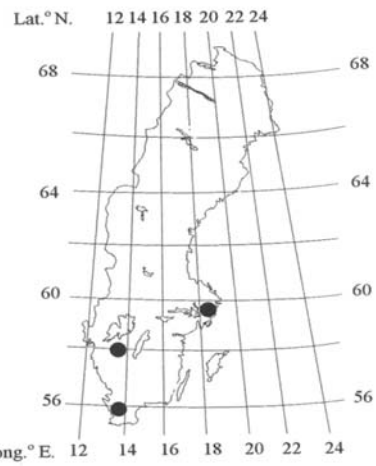

Fig. 1 Map of Sweden showing the locations of the three sampling areas.

The sampled trees were subjectively selected as the forest owner had restrictions about the future management of the stand. At each site one to four trees were selected for measurements, which the selection standards were healthy, undamaged, with fairly straight, single stems, and neither border trees nor suppressed trees. Generally, the selected trees had a DBH between the arithmetic mean DBH and the mean basal area-weighted DBH. In total, 69 trees were sampled for the stem-taper equation construction. For each tree the total height (m) was measured and the total age was defined by counting annual rings from a cut disc at stump height (0.2 m). A diameter (cm) on bark at breast height (1.3 m) and on the middle of 1 m-section of the stem was cross-callipered. According to the routines for yield studies at the Department of Energy and Technology, SLU, Uppsala diameters were cross-callipered at six relative heights of the tree (1, 10, 30, 50, 70, and 90%). Relative diameter and height points for the data set are shown in Fig. 2.

Table 1. Main characteristic on poplar stands.

Plot No.	No. of sample trees	Age, yrs	Dom. Height, m	DBH, cm Mean±SE	No. of stems·ha^{-1}	Basal area, m^2·ha^{-1}	Soil type
1	2	18	24.0	24.8±0.3	875	42	Light clay
2	1	41	27.0	33.4±1.4	973	87	Light clay
3	1	43	24.7	26.8±0.5	1906	107	Light clay
4	1	17	20.2	22.2±0.5	550	25	Medium clay
5	1	16	19.2	18.7±0.4	1111	30	Medium clay
6	3	21	29.2	33.0±0.5	361	30	Light clay
7	2	20	24.5	27.7±0.4	549	33	Light clay
8	4	23	22.8	19.6±0.7	632	19	Light clay
9	2	34	25.7	30.6±0.8	840	62	Light clay
10	2	20	24.5	26.7±0.6	520	29	Sandy silty till
11	4	16	20.2	12.8±0.8	3279	42	Light clay
12	1	18	21.1	20.0±0.5	909	29	Heavy clay
13	2	19	28.5	24.6±0.4	1250	59	Medium clay
14	1	19	18.5	24.0±0.6	295	13	Medium clay
15	1	34	27.2	29.1±0.8	398	27	Medium clay
16	2	24	25.9	29.3±0.3	457	31	Light clay
17	2	19	21.0	19.3±0.7	1111	32	Medium clay
18	2	20	21.6	18.2±0.7	1111	29	Medium clay
19	2	20	20.1	17.4±1.0	800	19	Medium clay
20	2	23	22.0	25.6±0.9	1005	52	Medium clay
21	2	20	22.5	23.6±0.7	1015	44	Medium clay
22	2	21	24.6	18.6±0.7	1200	33	Light clay
23	1	19	21.5	23.2±0.9	650	28	Light clay till
24	2	14	17.8	12.1±0.5	3493	40	Light clay
25	1	17	21.2	22.6±0.1	378	11	Sandy-Silty tills
26	3	21	29.1	28.3±0.9	506	32	Light clay tills
27	2	19	27.6	28.0±0.4	440	27	Light clay tills
28	1	20	29.5	25.1±0.3	707	35	Medium clay
29	2	17	24.8	29.8 ± 0.5	520	32	Light clay
30	2	21	28.0	31.0 ± 0.8	800	40	Light clay
31	2	20	27.0	27.2 ± 1.3	800	32	Light clay
32	2	18	26.0	27.3 ± 0.5	909	46	Medium clay
33	3	16	21.2	18.6 ± 0.5	966	26	Light clay
34	1	20	20.5	20.4 ± 0.7	1461	48	Medium clay
35	2	20	24.5	40.4 ± 0.8	198	23	Sandy-Silty tills
36	2	19	23.0	17.8 ± 1.1	2900	80	Light clay
37	1	20	27.0	34.9 ± 0.9	549	52	Light clay
Mean±SE		21±1.1	23.9±0.5	24.7 ± 1.0	984±125	38.5 ± 3.3	
Range		14-43	17.8-29.5	12.1-40.4	198-3493	11–107	

Stem taper equations tested

A polynomial equation was constructed and compared with five well-known stem taper equations (Table 2): a taper equation developed by Kozak et al. (1969) fitted for 19 species in British Columbia, Canada; a taper equation published by Ormerod (1973); a taper equation developed by Benbrahim and Gavaland (2003) for short rotation poplar plantations in France; a segmented taper equation developed by Max & Burkhart (1976) (modified version according to Sakici et al. 2008) for plantations

and natural stands of loblolly pine (*Pinus taeda* L.) and a variable exponent equation developed by Kozak (1988).

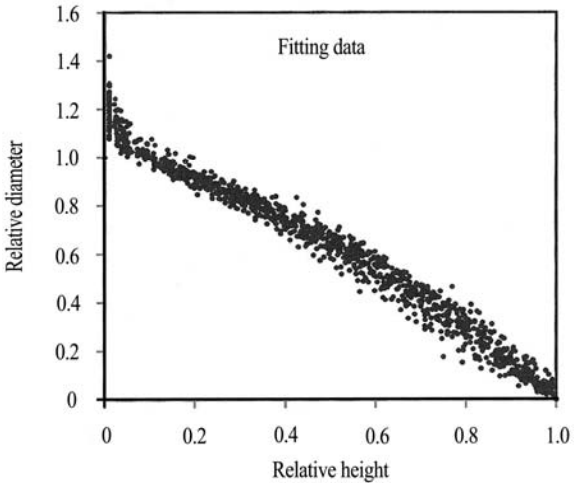

Fig. 2 Relative diameters and heights for the data set

Table 2. Six taper equations and their corresponding mathematical expression

Model	Expression	
Constructed polynomial equation	$d = (b_1q^2 - b_2q + b_3((H\text{-}h)/h)+b_4) \times (D/(1\text{-}k/H))^b5$	(1)
Kozak et al. (1969)	$(d/D)^2 = b_1 + b_2\, q + b_3(h^2/H^2)$	(2)
Ormerod (1973)	$d = D((H\text{-}h)/(H\text{-}k))^{b}{}_1$	(3)
Benbrahim and Gavaland (2003)	$d = Db - Db((ln\ (1\text{-}h/\ b_1H)/\text{-}\ b_2))^{1/b}3$	(4)
Max and Burkhart (1976)	$d^2 = D^2\,(b_1(q-1) + b_2(q^2-1) + b_3(a_1-q)^2\, I_1 + b_4(a_2 - q)^2\, I_2)$ I_1=1, if $q< a_1$; 0 otherwise I_2=1, if $q< a_2$; 0 otherwise	(5)
Kozak (1988, eq. 7)	$d = b_1\, D^{b2} b_3{}^D((1-q^{0.5})/(1\text{-}p^{0.5}))^A$ $A=(b_4q^2+b_5\ln(q+0.001)+b_6q^{0.5}+b_7e^q+b_8(D/H))$	(6)

where:
D = diameter at breast height, cm
Db = diameter at stump height, cm
d = stem diameter, cm, at height h
H = total height, m
h = height, m, from ground to top diameter (d)
$a_{i,}$, b_i = regression coefficients estimated from sample data
$q = h/H$, relative height
HI = height, m, of the inflection point from ground
$p = HI/H$
k = breast height (1.3 m)

Statistical analysis and procedure

The occurrence of multicollinearity and autocorrelation are the main problems when constructing taper equations, especially in models including complicated polynomial and cross-product terms (Kozak 1988). When severe multicollinearity occurs it could lead to the following problems:

(1) small changes in the dataset may produce significant changes in parameter estimates

(2) the regression coefficients have high standard errors

(3) the regression coefficient may have the wrong sign.

Ordinary least square method relies on the assumption that residual errors are independent and identically distributed. However, stem taper models are developed from data that is hierarchical in nature with several height measurements taken on the same individual tree bole. The data between the different points on the same trees are closely dependent on each other. This autocorrelation between the data points violates the above assumption of independence. According to Kozak (1997) autocorrelated error terms in a model can result in following consequences:

(1) the estimators no longer have the minimum variance property even though the regression coefficients are unbiased and consistent.
(2) the calculated mean squared error (MSE) may underestimate the real variance of the error terms, while the standard errors of the regression coefficients may underestimate the true standard deviation.
(3) statistical tests using t or F distributions and confidence intervals are no longer reliable.

The regression analysis was carried out using the SAS statistical package (SAS 2006). The NLIN procedure was used for fitting and developing the polynomial model and for estimating parameters and evaluating the six models considered in the study. The level of multicollinearity for the tested equations was identified by the PROC REG procedure. To identify the level of multicollinearity for the tested equations in the present study the condition index, CI (square root of the value of the largest and the smallest eigenvalue of the correlation ratios) was used. An indicator of serious multicollinearity is a CI >30 (Kozak 1997). AIC (Akaike Information Criteria) and BIC (Bayesian Information Criteria) are common goodness of fit criteria when comparing model with dataset affected by autocorrelation. According to Li and Weiskittel (2010) these criteria are not appropriate for selecting and comparing taper equations when the response variables between the equations are not the same. The response variable for equation (2) and (5) differ from the other equations. In some cases a slight change in curve shapes might be shown after a correction of autocorrelation. And the correction could hide the problem within the model, introducing misinterpretation of the observed trends (Nord-Larsen 2006). However, from a practical point of view the autocorrelation problem is generally ignored when using models for prediction of diameter and height (Diéguez-Aranda et al. 2006 a,b; Monserud 1984; Rayner 1991; Diaz-Maroto et al. 2010). In practice uncorrected equations could be used in forest management (Eriksson et al. 1997; Elfving & Kiviste 1997). A test of with-in test of autocorrelation for the equations in the present study showed that the models failed to achieve convergence. According to Yang et al. (2009) this is not surprising as within-subject could be difficult to justify. Other factors in the model affect the correlation (Fang and Bailey 2001). Based on these findings the autocorrelation was not corrected in the present study.

In order to determine how well the models fit the data, an

analysis of residual plots (Fig. 3) and the following statistics were used for the models and for data at different stem levels:

Coefficient of determination (R^2) = $1-\sum(d_i-\hat{d}_i)^2/(d_i-\bar{d}_i)^2$

Bias (B) = $\sum(\mathrm{Diff})/n$

Absolute Bias (AB) = $\sum|\mathrm{Diff}|/n$

Relative Absolute bias (AB %) = $100\times\sum|\mathrm{Diff}|/\sum_{i=1}^{n} d_i$

Sum of Squared Relative Residuals (SSRR) = $\sum(Diff/d_i)^2$

Root Mean Square Error (RMSE) = $\sqrt{Diff_i^2/(n-p)}$

where Diff = difference between observed and predicted diameters.

The sum of squared relative residuals (SSRR) is an important statistic in the analyses of differences between equations (Figueiredo-Filho et al. 1996). According to Parresol et al. (1987) absolute bias (AB) and SSRR provide a clear distinction between examined equations.

According to Huang et al. (2003) and Kozak & Kozak (2003) the validity of the models could be assessed by an independent data set. The number of available stands was sparse. Therefore the prediction of the model was done by a leave-one-out cross validation procedure. One observation was left at a time and the parameters were estimated for the reduced data set (Table 3).

Table 3. Estimated parameters and evaluation statistics of stem taper equations 1–6

Parameter	Parameter estimates	Standard errors of parameters	R^2	RMSE	B	AB	AB%	RMSE[1]
Equation (1) Constructed polynomial equation								
b_1	-0.4396	0.0198	0.996	1.20	0.04	0.86	5.2	1.20
b_2	0.8477	0.0250						
b_3	0.0020	0.0001						
b_4	1.2892	0.0247						
b_5	0.9130	0.0053						
Equation (2) Kozak et al. (1969)								
b_1	1.1578	0.0056	0.995	1.51	0.23	1.09	6.6	1.48
b_2	-2.1357	0.0182						
b_3	0.9690	0.0139						
Equation (3) Ormerod (1973)								
b_1	0.8593	0.0041	0.994	1.50	0.29	1.04	6.3	1.49
Equation (4) Benbrahim & Gavaland (2003)								
b_1	1.2984	0.0252	0.993	1.62	0.24	1.24	7.5	1.61
b_2	1.4444	0.0576						
b_3	1.5507	0.0201						
Equation (5)Max & Burkhart (1976)								
b_1	-2.9517	0.0448	0.996	1.31	0.19	0.94	5.7	1.31
b_2	1.4649	0.0261						
b_3	57.4125	2.7642						
b_4	-1.0902	0.0560						
a_1*	0.0820	0.0070						
a_2*	0.6500	0.0235						
Equation (6) Kozak (1988)								
b_1	0.7626	0.0451	0.998	0.91	-0.01	0.64	3.9	0.91
b_2	1.1054	0.0242						
b_3	0.9954	0.0008						
b_4	1.0144	0.1129						
b_5	-0.1997	0.0260						
b_6	1.2213	0.2444						
b_7	-0.6353	0.1331						
b_8	0.2491	0.0079						

* rel bind-point, 1) Leave-one-out cross validation

Results

All six taper equations had high correlation coefficients, R^2 = 0.99. The RMSE and AB values of the equation (6) were lowest among the equations, 0.91 and 0.64 respectively, indicating that it has good ability to predict stem taper. The relative absolute bias (AB%) value is 3.9% for equation (6) compare to the other equations with AB% values ranging from 5.2% to 7.5%. Equations (2), (3) and (4) had the highest RMSE values: 1.51, 1.50, 1.62 respectively and AB values 1.09, 1.04, 1.24 respectively. Bias (B) were close to zero for equations (1) and (6), indicating that the equations neither tends to under- nor overestimate poplar diameters, while the other equations have positive B, ranging from 0.19 to 0.29 indicating a slight underestimation. The parameter estimates and evaluation statistics of the studied equations are summarized in Table 3. The inflexion point (HI) in parameter p (HI/H) in equation (6) was found to be 10.2% of the total heights (H).

The results of AB for the relative heights on the fit data show that equation (6) has lowest values for the levels 10% to 90%. The SSRR for the levels 10% to 50% show minor difference between the equations while equation 6 had notable lower values for the upper part of the bole (70% and 90% relative height). Equations (4), (5) and (6) performed remarkable better at 1% relative level compare to equations (1), (2) and (3). Equations (4) to (6) had AB value between 1.02 and 1.18 and SSRR value ranges from 0.08 to 0.14. Equations (1), (2) and (3) have AB values of 1.43, 2.12 and 2.77, respectively, and the value of SSRR 0.28, 0.40 and 0.59 respectively. All equations show high SSRR values at 90%, where equation (6) have the lowest, 3.89 and equation (3) the highest, 9.87 (Table 4).

Equations (1) – (4) showed low levels of multicollinearity, CI ≤ 10. For equations (5) and (6) CI were close to 50 and 500 respectively, which are considered to be a level of severe multicollinearity (Kozak 1997).

Equations (1) and (4) did not met the zero criteria for predicted diameter at the top of the tree (h=H). The deviance was 0.2 and 0.4 cm respectively for the data. The residual plots show

that Equation (6) has a smaller residual distribution than the other equations, indicating the best ability to predict stem taper (Fig. 3). In the plot of residuals over relative heights the residuals are well balanced and distributed in an even manner for equation (6). The other equations are to some degree unbalanced where equations (2) to (4) are slightly more unbalanced than equations (1) and (5) (Fig. 3). The leave-one-out cross validation procedure exhibits small decreased RMSE values (< 0.03, Table 3).

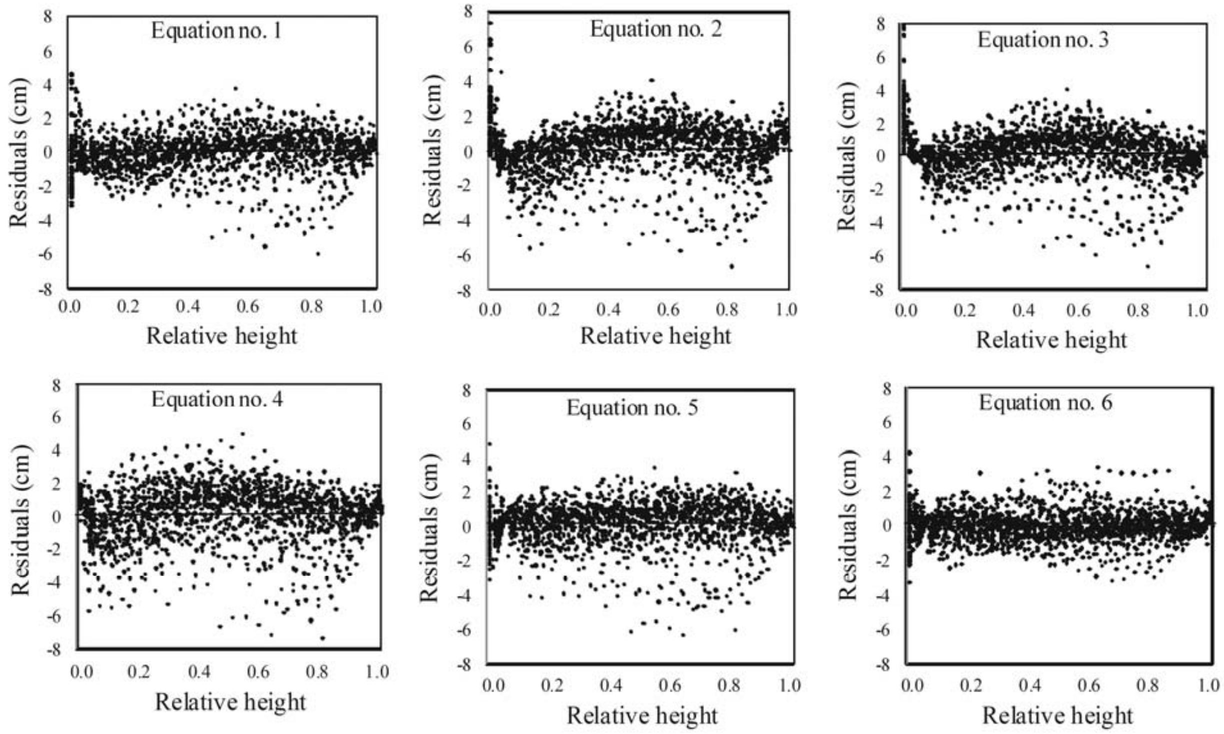

Fig. 3 Residuals for observed diameters against relative heights

Table 4. Statistics of fit for seven relative heights and the breast height (1.3 m)

Models		Relative heights							
		1	1.3 m	10	30	50	70	90	Mean
	B	-0.58	-0.06	-0.45	0.11	0.33	0.47	0.21	0.03
(1)	AB	1.43	0.57	0.83	0.86	1.02	1.21	0.89	0.88
	SSRR	0.28	0.06	0.12	0.15	0.33	1.07	7.23	1.32
	B	2.10	-0.56	-0.97	0.15	0.87	0.69	0.05	0.32
(2)	AB	2.12	0.57	1.05	0.99	1.36	1.44	0.93	1.05
	SSRR	0.40	0.03	0.15	0.20	0.62	1.52	9.68	1.80
	B	2.77	0.00	-0.50	0.26	0.78	0.62	0.10	0.54
(3)	AB	2.77	0.00	0.81	0.99	1.31	1.39	0.92	1.17
	SSRR	0.59	0.00	0.09	0.20	0.56	1.43	9.87	1.82
	B	1.10	-0.57	-0.62	0.61	0.89	0.50	0.08	0.28
(4)	AB	1.10	1.07	1.32	1.34	1.49	1.45	0.95	1.01
	SSRR	0.08	0.16	0.25	0.40	0.73	1.54	9.07	1.75
	B	-0.09	0.28	0.24	0.22	0.35	0.61	0.41	0.29
(5)	AB	1.02	0.47	0.81	0.95	1.10	1.39	1.05	0.97
	SSRR	0.09	0.03	0.07	0.15	0.31	1.06	6.35	1.15
	B	-0.13	-0.03	-0.10	-0.02	0.02	0.25	0.16	0.03
(6)	AB	1.18	0.23	0.68	0.72	0.82	0.91	0.68	0.71
	SSRR	0.14	0,01	0.07	0.11	0.25	0.67	3.89	0.74

(1) Hjelm, (2) Kozak (1969), (3) Ormerod (1973), (4) Benbrahim and Gavaland (2003), (5) Max and Burkhart (1976), (6) Kozak (1988). B, Bias; AB, Absolute Bias; SSRR, Sum of Squared Relative Residuals

Discussion

A number of stem taper prediction equations have been constructed for various species. However, prior to this study no stem taper equations had been developed specifically for poplar grown in Swedish or Scandinavian conditions. With increased interest in the use of poplar for biofuel and wood pulp in Sweden, the ability to predict yield using a stem taper equation will be of value. The systematic selection of sample trees used in this study should in general be avoided but was necessary by restrictions described in Material and Methods. According to Kozak (1997) subjectively selection of sample trees could cause the regression coefficients to be biased and variation appear smaller than its actual variation, compared to a random selection strategy. This is especially to be considered if the trees grow under various site conditions within a site and the range of tree size is wide. In the present study, however this problem is minor since all stands in the study are located on former farmland and the conditions within in the stands are homogenous and the range of tree size is small.

It has been suggested that there exist autocorrelation among the observations but the statistical theory tells us that the autocor-

relation should be between the errors within trees. Due to misspecification of the model, consecutive observations are probably correlated but that they should be autocorrelated is less clear. In fact our residual analysis does not indicate anything in this direction. What kind of inner product estimator one is using is probably less important for the mean estimate. An unweighted estimator is used due to non-linear least squares estimator has one fundamental advantage, namely the mean estimator will be independent of the dispersion estimator. For example, since our data are slightly skewed, the plain mean will usually not be affected much by non-symmetry, whereas the dispersion estimator is sensitive, leading to an unreliable weight.

Taper equations with a variable exponent which changes continuously along the stem (eg. neiloid root section, paraboloid mid section and cone-shaped top section) provide better predictions for the diameter from ground to the top of the tree than simple and segmented taper equations (Kozak 2004). Variable exponent taper equations have lower bias than other types of taper equations (Sakici et al. 2008). Analysis of the taper equations in the present study, based on the evaluation statistics confirms these findings.

Equation (4) developed by Benbrahim and Gavaland (2003) shows larger residuals based on data in the present study than in the 2003 study. The residuals in the 2003 study were <1 cm compared to residuals of up to 7 cm in the present study. The difference might depend on the difference in data structure. They used young stands (7–8 years) with a mean height of 13 m and a mean DBH of 12 cm. In the present study the stands are 21 years (range 14–43), and the dominant height and DBH is 24 m and 25 cm respectively. Benbrahim and Gavaland (2003) showed that no trend was observed in the residuals in their study. In the present study a trend was observed. Generally young poplars, as in the study of Benbrahim and Gavaland, have not developed butt-swells on the stems. The sampled older trees in the present study had distinct and developed butt-swells located from ground to 0.5 m of the stem. None of the studied equations could fully describe this butt-swell. Most of the large residuals for the simple equations are concentrated to this part of the bole. The presence of larger residuals located in the stump region is more pronounced for the simpler equations (2) and (3) compare to the other equations in this study (Fig. 3).

The poor performance for all equations at 90% of stem height is not important from a practical point of view (Figueiredo-Filho et al. 1996). The top part of the poplar (some meters below the top) is not for practical use except for bio-fuel. High level of multicollinearity was shown in equations (5) and (6), which may cause problems in analyzing the estimated coefficients (Kozak 2004).

Conclusion

When comparing the equations in the study, the variable exponent equation presented by Kozak (1988) showed the lowest absolute bias and AB% for all data levels along the whole trees (Table 3). AB for the different relative height levels and SSRR are in favor of the performance of equation (6) but equations (1 and 5) have low levels (Table 4). According to Parresol et al. (1987), AB and SSRR provide a clear distinction between examined equations and are important statistics when reaching to a conclusion and recommendation for a suitable equation to be used in practical surveys. The problem with severe multicollinearity for equation (6) does not seriously affect the prediction capability (Kozak 1997) and therefore not reason enough to reject the equation. Equation (6), which is highest ranked are recommended.

However when a less complex equation is requested and larger bias is accepted, the alternative equation to be recommended is partly depending upon the criteria of the diameter prediction at the top of the tree (h=H) since equation (1) and (4) did not meet the zero diameter prediction criteria. If a strict zero diameter prediction at top is not required, then the constructed polynomial equation (1) is alternatively recommended. Equation (1) is (together with equation (5)) ranked just after equation (6) regarding AB, SSRR and RMSE. If a strict zero diameter prediction at top (h=H) is required, the constructed polynomial equation (1) will be rejected as well as equation (4).

Acknowledgements

I want to express gratitude to my supervisor professor T. Johansson and to my assistant supervisor Dr. A. Karačić for their guidance. Special thanks to Professor Dietrich von Rosen and Associate Professor Sören Holm for their statistical guiding. Thanks to J. Bacenetti, M. Cardoso, L. Hedman, J. Johansson, Miss M. Johansson, M. Nilsson, and E. Temnerud, who helped me cutting and measuring the sample trees. Miss M. Johansson carried out the tree ring analyses in the laboratory and to Sees-Editing Ltd UK, which made the linguistic revision. The financial support was provided by Skogssällskapet foundation.

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